BULLETIN DE L'ACADÉMIE
11. POLONAISE DES SCIENCES
Série des sciences techniques
Volume XXII, No. 12 — 1974

QUANTUM ELECTRONICS

Influence of Parabolical Radial Gain Distribution of a Medium with Gain Saturation on Laser Modes. I

by

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Presented by R. SZEWALSKI on July 8, 1974

Summary. Approximate theory of a laser resonator filled with a non-linear active medium is presented under the assumption that the gain distribution over the transverse cross-section of the resonator is parabolic. The paper includes calculations for media having uniform transverse gain. The results suggest that the mode patterns are identical in both cases. It means that the influence of a parabolic radial gain distribution on the field distribution in the active confocal resonator is negligible within the gain range examined.

1. Introduction. Electromagnetic field distribution over mirrors of an active laser resonator and resonant frequency of such a resonator depend on its geometry and on parameters of an amplifying medium the resonator is filled with.

It is due to difficulties, mainly mathematical, resulting from taking into consideration the effect of propagation of radiation in a non-linear and non-uniform amplifying medium, that the problem of determination of field distribution and resonant frequencies of a laser resonator — the parameters of which determine a mode (type of oscillations) of the resonator — is usually confined to solving the problem for a passive resonator. This means that only the resonator geometry influence on the modes is taken into account whereas the influence of the amplifying medium is neglected [1]. Such simplification of the active laser resonator theory does not fully describe generating features of the laser resonator-active medium system.

The paper by Li and Skinner [2] presents one of the earliest approaches to the problem of a real active laser resonator. Nonuniformity of a linear amplifying medium transverse to the resonator axis was taken into account there. It was shown that in the case of small nonuniformities of the amplifying medium the modes of an active resonator under consideration differ but slightly from those of the corresponding passive resonator.

The further step into the theory of the active resonator was taken by Statz and Tang [3] who assumed non-linear features of the amplifying medium, resulting from the gain saturation due to the increase of radiation intensity within the resonator. Owing to mathematical difficulties involved an assumption was made that transversely uniform amplifying medium was concentrated in two infinitely thin sheets next to the surfaces of parallel-plane mirrors. In this model of the active resonator the radiation is propagating between the mirrors through a passive medium, whereas gain takes place directly on the mirrors. The above formulation of the problem of propagation and gain in a laser resonator simplified mathematics of the problem. The results obtained show that the modes of the resonator are negligibly affected by gain effects.

Fox and Li [4] extended the approximation of Statz and Tang onto the case of a parallel-plane and confocal resonators immersed in a transversely uniform non-linear amplifying medium. Their results of numerical calculations for a medium with small gain proved those basic properties of a mode, e.g. field distributions over the resonator mirrors, diffraction losses and resonant frequencies of the resonator are identical with those obtained for corresponding passive resonator. Locchi [5] obtained similar results for a parallel-plane resonator filled with transversely uniform non-linear amplifying medium.

Presented below is an approximate theory of a laser resonator filled with an active medium characterized by the parabolical gain distribution, with nonlinearity resulting from gain saturation taken into consideration. Parabolical gain distribution was chosen as an approximation of actual gain distributions in some laser media [6]. Also given are the results of numerical calculations for a confocal resonator filled with such an amplifying medium.

The aim of this paper was to obtain the information regarding simultaneous influence of a non-uniform gain distribution transverse to the resonator axis and a gain non-linearity caused by gain saturation due to increase of radiation intensity inside the resonator.

2. Mathematical formulation. The analysis will deal with a symmetrical laser resonator consisting of two identical spherical (or flat) mirrors Z_1 and Z_2 (Figure). The distance between them equals d. Radii of curvature are identical and equal to R. The mirrors are of circular shape with $2a$ diameter. It is assumed that all the resonator dimensions are large as compared to the wavelength of laser radiation and that the inequality

(1) $$a \ll d,$$

typical of laser resonators is valid. The whole space between the mirrors is filled with a medium, the gain coefficient of which $\alpha(r, w)$ is given by the formula [7]

(2) $$\alpha(r, w) = \frac{\alpha_0(r)}{\left(1+\dfrac{w}{w_0}\right)^{\nu}},$$

where $\alpha_0(r)$ is the unsaturated gain coefficient (for $w \ll w_0$), w is the radiation intensity and w_0 is a saturation parameter. The exponent ν is equal to $\frac{1}{2}$ or 1 for the inhomogeneously or homogeneously broadened line, respectively.

The gain coefficient of the medium $\alpha(r, w)$ is defined by the formula

(3) $$dw = \alpha w dz,$$